2007年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2007

学科試験　　問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

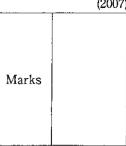COLLEGE OF TECHNOLOGY STUDENTS

数　　　学

MATHEMATICS

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)	Marks	

1 Fill in the blanks with correct numbers or expressions.

1) Solve the equation $8x^2 - 10x + 3 = 0$.

$$x = \boxed{① \qquad\qquad}$$

2) Solve the simultaneous inequality $\begin{cases} 2x^2 - 5x - 3 < 0 \\ 3x^2 - 4 - 11x \geq 0 \end{cases}$.

$$\boxed{② \qquad\qquad}$$

3) Solve the equation $2\cos^2 x + 3\sin x - 3 = 0 \quad (0° \leq x \leq 180°)$.

$$x = \boxed{③ \qquad\qquad}$$

4) Solve the inequality $\log_3 (2x - 1) + \log_3 (x - 1) < 1$.

$$\boxed{④ \qquad\qquad}$$

5) Let $\vec{a}, \vec{b}$ be vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$. Suppose $\vec{a} - \vec{b}$ be perpendicular to $6\vec{a} + \vec{b}$. Find the angle between $\vec{a}$ and $\vec{b}$.

$$\boxed{⑤ \qquad\qquad}$$

6) Suppose α and 3α are roots of the quadratic equation $3x^2 + 8x + k = 0$, where k is a real constant. Find the value of k .

$$k = \boxed{⑥ \qquad\qquad}$$

7) Let a be the integral part of $\dfrac{1}{2-\sqrt{3}}$ and b the decimal part $(0 < b < 1)$. Calculate the value of $a - b + \dfrac{2}{b}$.

$$a - b + \dfrac{2}{b} = \;⑦ \boxed{}$$

8) Let $\log_{10} A = a$ and $\log_{10} B = b$ for positive numbers A, B which are not equal to 1. Suppose $a + b = 0$. Find the value of $A^{\frac{1}{b}} B^{\frac{1}{a}}$.

$$A^{\frac{1}{b}} B^{\frac{1}{a}} = \;⑧ \boxed{}$$

9) Let $\sin\alpha + \cos\beta = -\dfrac{1}{2}$ and $\cos\alpha + \sin\beta = \dfrac{\sqrt{3}}{2}$ for $0 < \alpha < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < \beta < \pi$. Find the value of $\alpha + \beta$ by calculating $\sin(\alpha + \beta)$.

$$\alpha + \beta = \;⑨ \boxed{}$$

10) Let $a^{\frac{1}{2}} + a^{-\frac{1}{2}} = \sqrt{5}$. Find the value of $a + a^{-1}$.

$$a + a^{-1} = \;⑩ \boxed{}$$

11) Calculate $\displaystyle\sum_{n=1}^{\infty} \left(\dfrac{9}{10}\right)^n$.

$$⑪ \boxed{}$$

12) Calculate $\displaystyle\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$.

$$⑫ \boxed{}$$

13) Find the derivative $\dfrac{dy}{dx}$ of $y = (x+2)\sqrt{x}$.

$$\dfrac{dy}{dx} = \;⑬ \boxed{}$$

14) Calculate $\displaystyle\int_0^2 (x-1)^3\,dx + 2\int_{-1}^2 x(x-1)\,dx$.

$$⑭ \boxed{}$$

M— 2

2 Let $A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$, $E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

1) Find the values of p, q which satisfy $A^2 + pA + qE = 0$.

2) Find the values of a, b which satisfy $x^n = (x^2 + px + q) Q(x) + ax + b$ where $Q(x)$ is a polynomial and n is a positive integer.

3) Calculate A^n by substituting A for x and replacing E for 1 in
$$x^n = (x^2 + px + q \times 1)Q(x) + ax + b \times 1.$$

3 Consider the functions $f(x) = \min\{x^3 + 1,\ 3 - x\}$, $F(x) = \int_1^x f(t)\,dt$, where
$$\min\{p,\ q\} = \begin{cases} p & (\text{if } p \le q) \\ q & (\text{if } p > q) \end{cases}.$$

1) Sketch the graph of $y = f(x)$.

2) Find the maximal value and the minimal value of $F(x)$.